Dialogic Learning

From an educational concept to daily classroom teaching

Peter Gallin

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The development of the concept of self-controlled and sustainable learning is based on a personal encounter between two teachers of entirely different subjects. Two examples show how uncomplicated teaching mathematics in the classroom can be, once the teacher has gained the courage to trust in the capabilities of the children. The three textbooks “Ich-Du-Wir” (“I-You-We”) for German and mathematics in the first six years of elementary school provide support.

With our publication *Dialogisches Lernen in Sprache und Mathematik* (“Dialogic Learning in Language and Mathematics”) (Ruf & Gallin 2005) over ten years ago, Urs Ruf and I attempted to pool the wide variety of experience we had ourselves gained as *Gymnasium* (high school) teachers as well as that from colleagues of all school levels we met in our further training courses. Thus we tried to develop a uniform teaching concept we now call “dialogic learning”. These practical educational reflections, which extensively took place parallel to our teaching work at high school and remote from empirical educational research at universities, met with a satisfying response in German-speaking regions and have in the meantime become established in the scientific community. This was largely due to a shift in our focal points to the University of Zurich, which also freed our concept from the initial bond with the grammar school subjects of German and mathematics. Nonetheless, the essence of “Dialogic Learning” still focuses directly on practical classroom activities and on a realistic, efficient time and effort management for all persons involved in the lessons. To ensure that the concept can also be implemented at primary school level, we additionally developed “I-You-We” textbooks for German and mathematics for the first six years of school, which are used here and there as teaching aids officially approved in the canton of Zurich (Ruf & Gallin 1995; Gallin & Ruf 1999). This article will, on the one hand, present Dialogic Learning in a concise framework and, on the other, provide pointers to the – by its nature free – use of the “I-You-We” schoolbooks.

Genesis and theory of Dialogic Learning

Over many years, Dialogic Learning was developed through dialog in a constant process of critical analysis of classroom teaching practices. The foundation was laid in the 1970s in the framework of interdisciplinary cooperation at the Kantonsschule Zürcher Oberland in Wetzikon, Switzerland. Urs Ruf, a teacher and professor of German, and I, a mathematician, were looking for points our two subjects had in common. We quickly realized that although there are overlaps, they are not of primary importance for high school teaching. Our cooperation rapidly shifted to the basic problems that students repeatedly have to master in our school subjects. By a stroke of luck, it turned out that Urs still had lasting memories of his own mathematics lessons at high school – not all of them of a positive nature. As far as my German lessons at school were concerned, I had endured a similar experience. This constellation enabled us to analyze the process of learning in these two subjects without having to take into account common topics. Our interdisciplinary cooperation, which we then called “overlapping” instead of merely “touching”, was characterized by the following approach: Whenever we examined a topic involving either German or mathematics, the one who had majored in the subject took on the role of an expert, the other the role of a novice. In this way, the respective teacher had a student to deal with, who was interested in the unfamiliar subject and willing to learn, but was also able to clearly indicate and articulate his difficulties.

A concrete example from the beginning of our cooperation serves to illustrate how the didactic dialog between us took place. What you need to be aware of at this stage is that I have always had a special interest in games of logic and brainteasers ever since my university student days. At that time, I did not realize their didactic significance – in contrast to the didactic significance of the specified syllabus for mathematics. Intuitively, I liked confronting others with such problems because, as a general rule, the people concerned could not simply fall back on a formula or predefined procedure to solve the problems. One of the characteristics of brainteasers is, therefore, that they reveal the one-dimensional image of mathematics that many people have. They think that mathematics is a science that consists of exercises and questions for which a solution can always be found by means of formulas (algorithms) that have to be learned. Today, we call this restricted (one-dimensional) view of mathematics a “mathematical injury” (Fig. 1). Unfortunately, even today mathematics instruction rarely manages to convey a differentiated view of mathematics. This, however, is precisely the aim of Dialogic Learning in mathematics as a school subject.

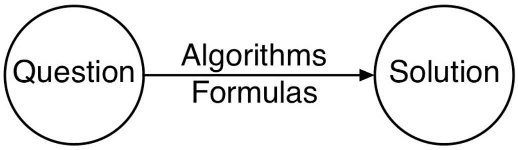

Fig. 1: A restricted (one-dimensional) view of mathematics: the mathematical injury

During our first didactic dialog, I was of course unaware that Urs had been made a victim of this mathematical injury in his former mathematics lessons, to the extent that he believed he had to answer every mathematical question immediately with a formula. This is why he felt great distress when I described an authentic problem I was faced with while filling the tank of my car. As he later admitted to me, his first inner reaction to my story was: “What algorithm, what formula do I have to use to solve the problem as quickly as possible?” But he didn’t let it show, of course. As a Germanist, he had learned that attack is the best form of defense. Consequently, he protested, “What you’re telling me here isn’t complete at all. To me it sounds like one of those word problems where the author struggles through a story, but doesn’t disclose the crucial part and beats around the bush. If he were to reveal it, the problem would no longer be of any interest.” When I denied having withheld any information, he retorted, “OK, I’ll prove it to you. I’ll write down everything you’ve told me or better yet: the way I have understood it.” No sooner said than done. When I read his text, I exclaimed, “Something is missing here!” It was, of course, a great triumph for him. “That’s exactly what I wanted to prove”, he answered. But I didn’t relent, didn’t reproach him and took a closer look at his text. I rewrote it and gave him the new version to read. Then he said, “Now I don’t understand the story anymore.” He rewrote the story again, after which it became my turn to declare, “Now the problem can no longer be solved.” The story went back and forth in this manner several times until we both agreed on the version that resulted from this written dialog. Satisfied with the text, we unfortunately threw away all the previous versions. Today it would be interesting to retrace this development process once again. At the time, we were, however, only interested in the final result, which was to become part of a small book we had decided to publish. For this booklet we jointly formulated fifty puzzles from my collection word by word, sentence by sentence, as in the first story. It was then published in 1981 by Silva-Verlag Zurich under the title *Neu entdeckte Rätselwelt* (“Newly Discovered World of Puzzles”) (Gallin & Ruf 1981). The first story described above was included as problem no. 17, which carried the title “While Filling the Tank.”

I had parked my car in front of one of the many gas pumps at a shopping center. A green light showed me that it was available for use. It was a self-service filling station. When a customer has finished pumping gasoline, a red light on the pump lights up showing that it is now blocked. The customer takes the receipt printed by the machine and goes to the cashier, who supervises the entire filling station. Once the customer has paid, the cashier unblocks the respective pump from a central control panel. When I lifted the nozzle, I noticed the display had already been reset to zero. I filled the tank, read off how much gasoline I had put in and took the receipt from the machine. Without taking a closer look at it, I went to the cashier, handed over the ticket and wanted to pay. The cashier then exclaimed: “Now it’s happened!” He went to the pump and came back with a receipt showing the right number of liters and the invoice in Swiss francs.
What was on the first receipt? Can you reconstruct the incident?

Intensive analysis and persistent formulation attempts enabled Urs repeatedly to come up with solutions for maths problems he was faced with. This happened almost incidentally, not because he had a formula to fall back on, but because he successfully thought his way through the situation underlying the problem. What took place here can be represented in our diagram by the additional “I”, which symbolizes the position of Urs (Fig. 2).

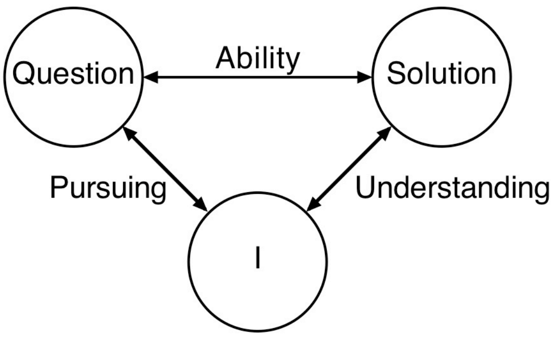

Fig. 2: Understanding is only possible if the “I” pursues thoroughly a question

Urs’ encounter with the problem has two characteristic features:
1. The “I” of the learner was evidently activated by my provoking question, and
2. Urs was able to get a hold on the problem through his spontaneous writing.

We call the interaction between question and I “pursuing mathematics”. Initially, the focus is thus not on solving the problem, but on exploring the question and related aspects at depth until the question becomes a genuine question for the student himself/herself. It is a well-known fact that parroting the wording of a question by no means constitutes a real question that students would actually ask themselves. So in the process of pursuing mathematics, you literally ignore the solution. And a decisive point for us here is that you speak or write in your own language, your native language or – as Martin Wagenschein calls it – the language of understanding, not in some technical jargon or in the arcane lingo of insiders, of people who have already understood (Wagenschein 1980). It has often been my experience that intensively “pursuing” mathematics leads students to the solution without their even becoming aware of it. It was the same with Urs. I had to tell him several times that he had already found the solution and could stop turning over the question in his mind, pursuing mathematics. It turned out that as a linguistic expert something completely different fascinated him from what I had anticipated on the basis of my own subject-related expectations: it was not the concentrated, unambiguous, and apodictic solutions for our problems, but the entire mathematical landscape surrounding the problem that actually captivated Urs. What interests him is how to successfully relate the question to one’s own world and to make the most of various approaches to the mathematical result. The third link in our diagram, the "understanding of mathematics", is thus generated quasi automatically if mathematics has been pursued long enough. This experience is supported by a statement made by philosopher Hans-Georg Gadamer, in which he specifies a necessary condition for understanding: "The very first stage in the process of understanding is when something appeals to us: that is the paramount of all hermeneutic conditions." (Gadamer 1959)

Understanding is never in the hands of the teacher. You cannot get someone to understand, all you can do is try to increase the probability that the student will feel the "appeal", as Gadamer puts it. Understanding always comes about unexpectedly, it cannot be planned and organized. Physicist Martin Wagenschein also asked himself how understanding comes into being and made the following observation: "Real understanding is brought about by talking to others: based on and stimulated by something enigmatic, looking for the reason." (Wagenschein 1986) For him, too, it all starts with a person's consternation over an "enigma." But an additional factor comes into play here, i.e. an exchange with other people who have also given thought to the same problem. This aspect has not yet been taken into account in our diagram, which is why we extend it to include a fourth position – the "You". This was the role I played in the dialog with Urs by responding to the solutions he tentatively suggested and raising new "questions" in him through my reactions. The dialog that develops between an I and a You in the learning process via the questions and solutions for a problem is made graphically visible as follows (Fig. 3).

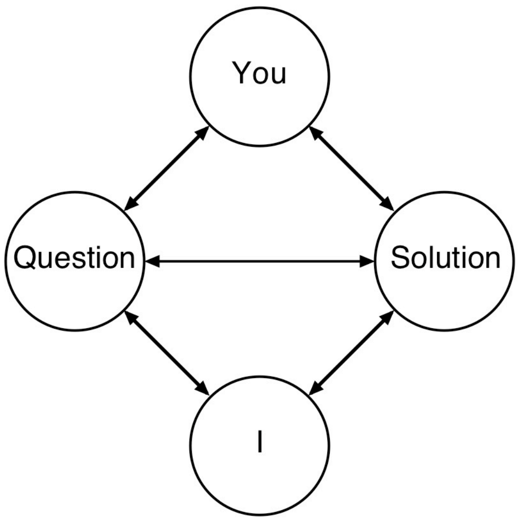

Fig. 3: The dialog during learning

Now the one-dimensional classroom instruction, which is solely limited to teaching formulas and algorithms (horizontal direction), has become a two-dimensional form of teaching, which includes the vertical dimension between the positions I and You. The connections between the two positions opposite each other intersect at a point that we designate as "We." That is where the regular perceptions of science meet the singular insights that develop in the dialog between the I and the You (Fig. 4).

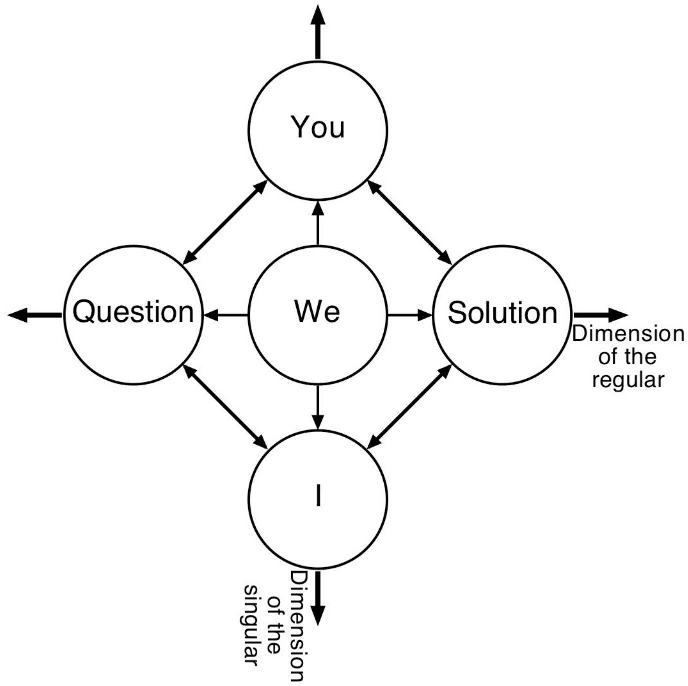

Fig. 4: Two-dimensional teaching

I would like to make it mandatory for extended forms of teaching, which are greatly recommended nowadays, to incorporate this feature of two-dimensionality. To me, they are beneficial only if the dimension of the singular (added to the regular) is actually brought into play. It is, after all, very possible to organize modern methodological arrangements in which only the dimension of the regular still counts. Genuine extended teaching therefore means classroom activities in which an exchange or dialog between an I and a You aimed at negotiating and defining the established regularities of the subject plays a major role, reaching all the way to the assessment and the awarding of grades.[1] Consequently students have the opportunity at school to find out how all formulas, norms, prescriptions, rules, and algorithms that exist – not only in mathematics – are, in the end, the result of a dialog, i.e. represent binding rules as a negotiated We position. Let us be clear in our minds about the fact that particularly in science all norms and substantiated results are ultimately the outcome of a dialog, an agreement among experts.

By renaming the five positions in Fig. 4, which is inspired by the individual learning and research situation, a final illustration will now show how teaching entire classes can be set up. At the same time, methodological references emerge that are typical of Dialogic Learning. At the beginning, there is not simply a question in its question form, but a provocation that induces the student to act on the factual level by means of an assignment. We call this the core idea. Through this core idea, the question is presented in a compact, attractive, and perhaps even provocative manner. The core idea is the guideline for preparing an assignment directed at all " I's" in the class. To make it possible to handle an entire class with all its heterogeneity, students are instructed to record the steps they take in tackling the assignment (learning journal or "travel diary"). These are the students' tentative "solutions" that are read by a You. Frequently this will be the teacher, but it is also entirely conceivable that other students might take a look at it beforehand and comment on the work others have done in their journals (by leaving the journals and changing places with others).[2] A decisive factor for Dialogic Learning is that the You provides (brief) feedback and thus acknowledges the students' core ideas that were actually effective in handling the assignment. It is

[1] See "Dialogisches Lernen in Sprache und Mathematik" ["Dialogic Learning in Language and Mathematics"] (Ruf & Gallin 2005, Vol. 2, p. 81ff.)
[2] See "Dialogisches Lernen in Sprache und Mathematik" (Ruf & Gallin 2005, Vol. 1, p. 39ff.)

perfectly possible for the students' core ideas to differ from that originally stated by the teacher. Further lessons receive new impetus from a suitable selection of the ideas found in the students' notes and a discussion of these ideas in the entire class. In Dialogic Learning, the norms that ultimately have to be learned in the subject concerned are hinted at rather than spelled out: they correspond to the We position, i.e. the target (crossing) between an I and a You at the end of the exchange.

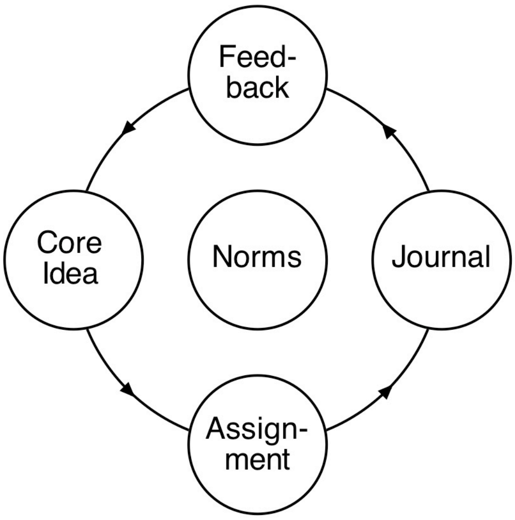

Fig. 5: The cycle of Dialogic Learning

Working with I-You-We as a teaching aid

The outline of the above theory and the numerous specifications involved in Dialogic Learning may put off teachers and make them think that superhuman powers are needed to meet all these requirements.[3] For this reason I, would like to use an example to provide suggestions for putting Dialogic Learning into practice with the help of the textbook "I-You-We" as a teaching aid and try to show that significant results can be achieved even with very small steps.[4] The first contact with multiplication is involved here and we follow the stages in the cycle shown in Fig. 5.

1. Core idea

A rather broad definition of the core idea states that "Core ideas have to be phrased in such a way that they arouse questions in the singular world of the student, which in turn direct attention to a certain subject area of the lesson." (Gallin & Ruf 1990, p. 37) The crucial element of a core idea is thus its effect on the student; it triggers productivity. In this function, therefore, a verbal form of a "core idea" is, strictly speaking, initially just a "candidate for a core idea" since its effect has yet to manifest itself in a specific lesson. Consequently, core ideas cannot be designated as such until later and then only in relation to a certain unique group of students. In addition to this, however, there are core ideas of teachers that have already demonstrated their effectiveness based on the particular biography and genesis of knowledge among the persons involved. And a large number of such core ideas – the core ideas of the authors and their acquaintances – are incorporated into the textbook I-You-We and expressed both in the main text and in the titles of the chapters and assignments. All of them relate to the official syllabus in the subjects German and mathematics and are intended to

[3] A detailed description of the approaches to a dialogic structure of classroom teaching can be found in my contribution in the book "Besser Lernen im Dialog" ["Learning Better in a Dialog"] (Gallin 2008a).

[4] Another detailed example from a 6th grade class [6. Primarklasse] of Patrick Kolb in Steinhausen can be found in my contribution on the rule of three in the book "Besser Lernen im Dialog" ["Learning Better in a Dialog"] (Gallin 2008b).

stimulate the students to grapple independently with the problem at hand via the assignments. In this way, it becomes clear what didactic role the puzzles that I employed to challenge others played: with a puzzle there is justified hope that it will act like a core idea and set a productive process in motion in the person confronted with the puzzle. Since it is not possible, however, to compress an entire syllabus into puzzles, core ideas have to take over this role.

An example that can be used in this context is the introduction of multiplication of natural numbers in the first years of school. Two core ideas are offered for this in I-You-We 1 2 3 (Ruf & Gallin 1995, p. 62ff.). The first one states: “Inner images help you to group a large number of similar objects clearly without having to touch them.” The second one is: “When you put on the multiplication glasses, you see multiplication calculations all around you.” It is highly unlikely that any core idea candidate will unfold its effect in school in this abstract form. For this reason, core ideas have to be transformed into specific assignments given to the students as mandatory tasks.

2. Assignment
The teaching aid always makes an initial suggestion for an assignment that is divided into several stages and becomes increasingly complex. Practice has shown that it is advantageous to hand out only part of the assignment at a time, either as a copy or by dictating it, so it is noted in the journal (diary) immediately prior to being worked on by the school children. This makes reading easier, particularly at a later stage and for third parties. The first part of the assignment “Multiplication glasses” is: “Imagine that you are wearing multiplication glasses and look around a bit in your environment. Do you discover things that are arranged nicely in groups of twos, threes, fours or fives? Make a note of them in your diary and write an appropriate multiplication calculation for them.” By making cardboard “multiplication glasses” with two round, empty holes for each child, the teacher creates an amusing way of giving the glasses a concrete function and thus makes it easier for the schoolchildren to fully dedicate themselves to the task at hand.

3. Journal
The journal excerpts shown in the textbook are intended to encourage teachers and students to try a similar approach. We weren’t able to predict this reliably, but surprisingly it doesn’t cross the children’s mind at all to copy these illustrations. They are evidently designed so personally that a natural inhibition keeps the pupils from copying them. The following example (Fig. 6) of eight-year-old Joana, which took place recently in a normal class in Zürich-Nord at the end of the first year in primary school, shows at a glance, despite the few words used, that Joana has already understood the nature of multiplications. Thanks to these clues, you can, so to speak, look into the child’s mind! A decisive element is the fact that the teacher has the courage to expect something from the children and doesn’t think she has to spell out everything herself in advance by handing out restrictive worksheets, for instance. The children in this class work using sketch pads, which additionally support the freedom of the individual product through their own lack of structure.

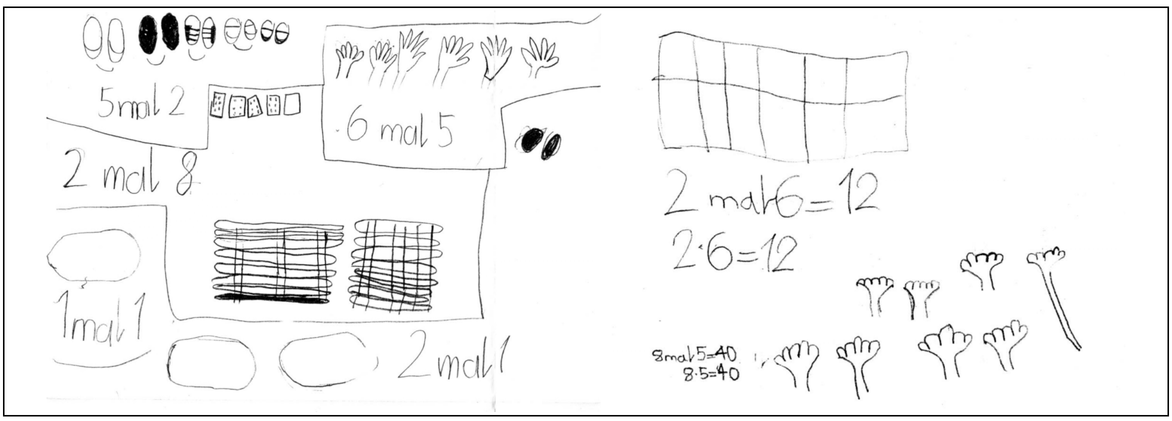

Fig. 6: With her pencil drawings Joana shows all the places where she sees multiplication calculations

(the German “mal” means “times”)

4. Feedback
Joana is justifiably proud of the fact that her teacher distributes her mature work in the learning journal and talks about it during a lesson. This acts as an incentive and creates a situation in which, sooner or later, even weaker students show above-average achievement relative to their standard so that their work can be discussed and appreciated within their class. However, this is only one aspect that plays a role in going through the students’ work again in class. Besides that, there is always an educational aspect that is decisive for the continuation of the lesson. In all the students’ works, there are one or more core ideas that can be extracted by the teacher and turned into a new assignment. As a result, the role of the first core idea given by the teacher or the textbook fades into insignificance. Furthermore, preparation for the following lesson can be carried out in the course of looking through the students’ works. Core ideas in Joana’s work may include: “It is useful not to state the result of the multiplication calculation right away” or “Pictures without calculations or with a mistake may give rise to a new puzzle”. New assignments could be formed on this basis and then be given to everyone in the entire class. This means, however, leaving the line of approach of the textbook for a moment, which is precisely the characteristic feature of Dialogic Learning. This kind of classroom teaching cannot be planned in detail, it develops from the contributions of the students. At the same time, the inherent problem of heterogeneity is tackled via this approach, because all children in the class repeatedly receive the same assignments, which they work on individually, albeit at varying depths and levels of intensity. Nevertheless, an exchange within a class is possible, the children can help one another and discuss things so that individualization does not lead to isolation. Instead, it leads to social learning within subject lessons.

5. Norms
A question that repeatedly arises is whether the specified teaching goals, norms, and competencies can be achieved through Dialogic Learning. Specifically, a question frequently asked is whether subject-related topics can also be practiced and tested. To the extent that the work in the learning journals is not in itself practice enough – Joana has already practiced several multiplication calculations – preparing for a test can itself be transformed into an assignment. The somewhat superficial, though often very effective core idea behind this is: “I want to get a good grade.” Accordingly, the teacher can give each child the assignment of inventing a problem that is as difficult as possible but nonetheless manageable and interesting at the current class level. And before you know it, the teacher is in possession of more than twenty problems that exert a very particular attraction for the students, in contrast to copies of predefined tasks. The authors are known, and the students are not even certain whether all problems are well-defined and solvable. Interesting subject-related discussions among the students are inevitable. Fig. 7 shows an example from my own teaching at the Kantonsschule Zürcher Oberland in Wetzikon, Switzerland.

881101StudentsProblemsU1b.doc Peter Gallin

Exercises relating to the distributive law, U1b, 1 November 1988

1.	Factor out: gcdbaef + hjkmci – oqsrcnp + zyxcwtuv	
2.	Calculate as elegantly as possible: $9738659667 \cdot 9738659967 - 9738659567 \cdot 9738659667 +$ $9738659667 \cdot 9738659467 + 973865966 \cdot 9738659267 - 9738659967 \cdot 9738659667$	
3.	Multiply out: $(a-b) \cdot (a-b) \cdot (a+b)$	
4.	Factor out: $a^3 + a \cdot a \cdot b - a \cdot b \cdot a - a \cdot b \cdot b + b \cdot a \cdot a + b \cdot a \cdot b - b \cdot b \cdot a - b^3$	Claudio
5.	Multiply out: $(f+x) \cdot (u-w)$	Anatina
6.	Depict in a drawing $(49 \cdot 5)$, with different colors as far as possible	
7.	Write as simply as possible: $7a^2 - (3a^2 - a)$	Christian
8.	Factor out and write as simply as possible: $c^2 \cdot a - b + c^3 \cdot 2c + b^2 \cdot c^2$	
9.	Calculate in the simplest possible way: $243378 \cdot 243379 - 243377 \cdot 243378$	Martina
10.	Which number has to be inserted for the placeholder so that the number pairs have the same quotient? (x^4,x^2), $(49^4,x^2)$	Sara
11.	Factor out the biggest possible factor: $4^3 + 4^2 + 4^5$	Sandra
12.	Calculate as simply as possible: $189357389562 \cdot 189359389562 - 189358389562 \cdot 189357389562$	
13.	Calculate $(a:(m+n))(m+n)$	
14.	Calculate simply $24 \cdot 89 + 53 \cdot 119 - 36 \cdot 24 - 43 \cdot 53$	
15.	Factor out: adam + eve - apple	
16.	Factor out: abcdefgh + bcdefghik - deghiklm	Kaspar
17.	Multiply out: $(a-b+c) \cdot (d+e) \cdot (f-g-h+l)$	Claudius
18.	$30003 \cdot 100 \cdot 29997 : 25 : 4$	
19.	$17985 \cdot 17985 \cdot 1398 - 1397 \cdot 17985$	Renate
20.	Multiply out: $(a+b-c) \cdot (a-b-c)$	Christof
21.	Multiply out: $(a^6 \cdot a^3 \cdot a^7 - a^8) \cdot (z^7 + 6^8 \cdot 9^9 - r^3) \cdot (v^4 + v^3 - v^2 \cdot 5)$	Sämi
22.	Factor out the biggest possible factor: $133 + u - 95 \cdot v + 38 : w \cdot 171 : x + 76 - y \cdot 114 - z$	Oliver
23.	Factorising is more difficult if the factor first has to be prepared separately: $95 \cdot p + 57 \cdot q^2$	Daniel
24.	Factor out: $279 \cdot a - 31 \cdot a \cdot b + 93 \cdot a^2$	Bettina
25.	Multiply out: $(a - b) \cdot (c - d)$	Katharina
26.	Factor out: $171 \cdot 256 - 114 \cdot 8^2$	Claudia
27.	Factor out the biggest possible factor: $396 \cdot x - 18 \cdot x \cdot y + 66 \cdot x^2$	Yvonne
28.	Factor out the biggest possible factor: $x \cdot 189 + y \cdot b \cdot 147 - a \cdot 105 + z \cdot 126$	Mike
29.	Factor out: $3^2 \cdot 3^2 - 9$	Anja
30.	The two number pairs have to have the same quotient: $(x^4,25^2),(x^4,625)$	Oli
31.	Simplify as far as possible: $z \cdot ((z^4 + z^3) : (z^3 + z^2))$	
32.	Factor out: $z \cdot z + z : z - z$	Röbi
33.	Calculate with the distributive law in the simplest possible way. A calculator with 8 digits is available: $45626809112100 \cdot 111125709813245$	Nadine
34.	Factor out: $(a^3 \cdot b)^2 - a^3 \cdot b^2$	Philipp

Fig. 7: Exercises regarding the distributive law, worked out by all students of a Gymnasium class (7th year of school)

In summary, we can state that it is possible to transform traditional classroom teaching into dialogic learning by means of three simple measures:

1. Believe in your students, and do not inundate them with prepared material.
2. Teach students to produce their own individual ideas on a central topic of the lesson.
3. Look through all journals of the students, select useful material, and only then continue with the next lesson.

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